12.3.2 Detection Limit of the Beam Deflection Method

Up to now we have analyzed the magnitude of the photocurrent I as a function of the external conditions such as deflection of the tip, laser power, wavelength, and the geometrical parameters of the setup. In the following, the detection limit for the optical beam detection, i.e. the minimum detectable deflection Δz of the cantilever, will be analyzed. The fundamental source of noise in the beam deflection scheme is shot noise, which arises due to the discrete arrival of the photons at the photodiode. Correspondingly, the noise of the electric current in the photodiode is induced by discrete number of electrons, each generated by a photon with a probability given by the quantum efficiency (generated electrons per photon at the respective wavelength). Here we use the sensitivity of the photodiode R defined above as an equivalent quantity. If we consider the optical power S_A irradiating segment A of the photodiode, the corresponding generated current is $I_A = RS_A$.

In the following, we estimate the fundamental limit in the noise of the photo current imposed by the discrete number of electrons (shot noise). An expression of this shot noise can be derived if one considers an electrical current occurring due to a discrete number of charges, N, flowing per time of measurement, Δt. If we allow for a long measurement time (averaging), say a second or so, the current will be measured with low noise, but this also means that for instance the AFM feedback can only run at this slow speed. Usually the speed of the measurement is expressed by the bandwidth, which is roughly the maximum frequency at which a signal can be detected properly, i.e. without too much loss of signal. If the duration of the measurement of the current is one second, the bandwidth is about one hertz. If the measurement bandwidth is defined as $B = 1/\Delta t$, the measured current generated by segment A of the photo diode I_A can be written as

$$I_A = eN\frac{1}{\Delta t} = eBN. \qquad (12.10)$$

If the current corresponds to N charges flowing by in the time Δt, the number of these charges will fluctuate on average by $\sqrt{N}$, leading to a current fluctuation of

$$\Delta I_{\text{shot},A} = eB\sqrt{N} = eB\sqrt{\frac{I_A}{eB}} = \sqrt{eI_A B}. \tag{12.11}$$

In our simplified explanation, a numerical factor of $\sqrt{2}$ is missing. In a statistically more rigorous derivation the following equation for the shot noise of segment A results

$$\Delta I_{\text{shot},A} = \sqrt{2eI_A B}. \tag{12.12}$$

The total current noise from both segments of the photo diode results as

$$\Delta I_{\text{shot}} = \sqrt{\Delta I^2_{\text{shot},A} + \Delta I^2_{\text{shot},B}} = \sqrt{2e(I_A + I_B)B} = \sqrt{2eRS_0 B}, \tag{12.13}$$

with $I_A + I_B = R(S_A + S_B) = RS_0$.

Identifying the photocurrent estimated above in (12.9) as signal S and the shot noise from (12.13) as the corresponding noise N, the signal-to-noise ratio is given by

$$\frac{S}{N} = \frac{I}{\Delta I_{\text{shot}}} = \frac{6d}{l\lambda} S_0 R \Delta z \frac{1}{\sqrt{2eRS_0 B}}. \tag{12.14}$$

The smallest detectable cantilever displacement results as

$$\Delta z = \frac{l\lambda}{6d} \frac{S}{N} \sqrt{\frac{2eB}{RS_0}}, \tag{12.15}$$

Now we discuss the dependence of the smallest detectable cantilever displacement on the different quantities involved. A laser beam with higher intensity S_0 will improve the detection sensitivity towards smaller Δz, however this will also pump more energy into the system which can lead to thermal drift and is especially undesirable in low temperature applications. With a larger measurement bandwidth B, i.e. a shorter averaging time for the measurement, the smallest measurable deflection Δz becomes larger. S/N is the signal-to-noise ratio at which a certain feature (for instance an atomic protrusion) can be just identified. If a signal strength of one, two, or three times the noise signal is required to distinguish a signal feature from noise, the smallest detectable height of that feature Δz will increase by one, two, or three times. In this sense, the smallest detectable cantilever displacement is proportional to the signal-to-noise ratio desired in order to resolve a feature. With a larger width d of the reflected spot on the back of the cantilever, the diffraction becomes less pronounced and therefore the sensitivity increases. However, the size of the deflected beam is limited by the cantilever width. With a smaller wavelength of the laser beam, the width of the diffracted beam becomes narrower and the sensitivity increases.